

**Math 444 - Homework 13****Name:** \_\_\_\_\_

1. Find the harmonic conjugate of  $u(x, y) = x^2 - y^2$ . That is, find a harmonic function  $v(x, y)$  such that  $u + iv$  is holomorphic.
2. Prove that if  $u(x, y)$  is harmonic and bounded on  $\mathbb{R}^2$ , then  $u(x, y)$  is constant. Hint: Recall that  $u$  must have a harmonic conjugate  $v$ . Use Liouville's theorem on  $\exp(u + iv)$ .
3. Give an example to show that the product of two harmonic functions is not necessarily harmonic.
4. Use the Residue theorem to evaluate the following real integrals.

(a) 
$$\int_{-\infty}^{\infty} \frac{1}{(x^2 + 1)(x^2 + 2x + 2)} dx$$

(b) 
$$\int_{-\infty}^{\infty} \frac{\cos x}{x^2 + 16} dx$$