Math 444 - Homework 3 **Name:** ______________________

1. Let a_n and b_n be sequences in $\mathbb{C}$. If a_n converges to zero and b_n is bounded, prove that the sequence $a_n \cdot b_n$ converges to zero.

2. If a_n converges to $a \in \mathbb{C}$ and b_n converges to $b \in \mathbb{C}$, then prove that $a_n \cdot b_n$ converges to $a \cdot b$. Hint: $a_n \cdot b_n - a \cdot b = (a_n - a) \cdot b_n + a \cdot (b_n - b)$.

3. Let $A \subseteq \mathbb{C}$. We can define a relation $\sim$ on A where $z \sim w$ if there is a (continuous) path $\gamma : [a, b] \to A$ such that $\gamma(a) = z$ and $\gamma(b) = w$. Prove that $\sim$ is an equivalence relation on A. Recall that to prove that a relation is an equivalence relation, you need to show that it is

   (a) Reflexive: $z \sim z$ for all $z \in A$.

   (b) Symmetric: If $z \sim w$, then $w \sim z$.

   (c) Transitive: If $u \sim v$ and $v \sim w$, then $u \sim w$.

4. Let $f(z) = z^2$. Sketch a graph of the curve $f(2+ti)$ for $t \in \mathbb{R}$. Be sure to include any points where the curve intersects the real or imaginary axes on your graph. Hint: Simplify $f(2+ti)$ before you try to graph it.

5. Suppose that $A_1, \ldots, A_n$ are open sets in $\mathbb{C}$. Prove that the intersection $\bigcap_{1 \le k \le n} A_k$ is an open set.

6. Give an example of an infinite collection of open sets $A_1, A_2, \ldots$ in $\mathbb{C}$ such that $\bigcap_{k \ge 1} A_k$ is not an open set.

7. Evaluate $\displaystyle\sum_{n \ge 1} \left(\frac{1+i}{2}\right)^n$ and simplify your answer.