Math 444 - Homework 6 **Name:** ______________________

1. Let $g(z) = \dfrac{z-i}{2iz+4}$. Find

   (a) $g(i)$

   (b) $g(0)$

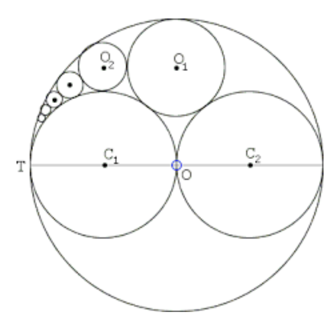
   (c) $g(\infty)$

   (d) $g(2i)$

2. Show that the Möbius transformation $f(z) = \dfrac{1+z}{1-z}$ maps the unit circle (minus the point $z = 1$) onto the imaginary axis. Hint: if you transform any three points on a circle, that will determine which circle or line you get.

3. Construct a Möbius transform $f(z)$ that sends -1 to infinity, but 0 and 1 are fixed points (i.e., $f(0) = 0$ and $f(1) = 1$). Hint: First find a Möbius transform that sends -1 to infinity and 0 to 0, then rotate/scale until 1 maps to 1.

4. Draw a picture to show how the Möbius transform above (in Problem 3) will transform this shape below (where T is the point -1 and O is the origin. Which circles become lines, and which stay circles?



The complex sine and cosine are defined by the formulas

$$\cos z = \frac{e^{iz} + e^{-iz}}{2} \text{ and } \sin z = \frac{e^{iz} - e^{-iz}}{2i}.$$

5. Show that these formulas agree with the usual sine and cosine when z is a real number.

6. When z is a real number, $\cos z$ and $\sin z$ are always bounded between 1 and -1. This isn't true for complex numbers. Find a formula for $\sin(iy)$ for any real number y and then show that

$$\lim_{y \to \infty} \sin(iy) = \infty.$$

7. Find all solutions of the equation $\sin z = 2$. Hint: Let $u = e^{iz}$. Then $\sin z = \dfrac{u - u^{-1}}{2i} = 2$. If you multiply both sides of this equation by $2iu$, then you get a quadratic polynomial, which has two solutions for u. Each of those correspond to a set of solutions for z.

Convert the following to rectangular form.

8. $e^{\sin(i)}$

9. $\mathrm{Log}(1 + \sqrt{3}i)$

10. $\mathrm{Log}\left(\dfrac{1}{3+4i}\right)$