

**Math 444 - Homework 7****Name:** \_\_\_\_\_

1. In this problem you will evaluate the integral  $\int_{\gamma} |z|^2 dz$  where  $\gamma(t)$  is the parabola  $\gamma(t) = -t + i(t^2 - 1)$  from  $t = -1$  to  $t = 1$ .

(a) What are the real and imaginary parts of  $|\gamma(t)|^2 \cdot \gamma'(t)$ ?

(b) Use the real and imaginary parts above to evaluate  $\int_{\gamma} |z|^2 dz$ .

2. Integrate the function  $z - \bar{z}$  on the upper half of the unit circle from  $z = 1$  to  $z = -1$ .

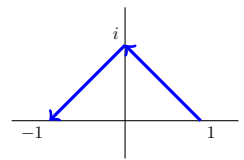
3. Find the length of the path  $\gamma(t) = t + \frac{2}{3}t^{3/2}i$ ,  $0 \leq t \leq 3$ .

4. Show that  $\lim_{n \rightarrow \infty} \left| \int_{\gamma} \frac{1}{z} dz \right| = 0$  when  $\gamma$  is the horizontal line segment from  $1 - ni$  to  $-1 - ni$ . Hint: One way to do this would be to calculate the integral exactly for any  $n$ . An easier alternative is to use the inequality

$$\left| \int_{\gamma} f(z) dz \right| \leq \text{length}(\gamma) \cdot \max_{z \in \text{range}(\gamma)} |f(z)|.$$

Use a computer (I recommend Sympy) to calculate the following integrals.

5.  $\int_{\gamma} (\bar{z})^3 dz$  on the piecewise path shown below. Hint: in Sympy the complex conjugate function is `conjugate( )`. You'll need to parameterize each piece separately.



6.  $\int_{\gamma} (\bar{z})^3 dz$  on the piecewise path shown below.

