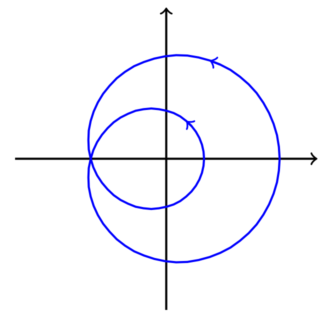
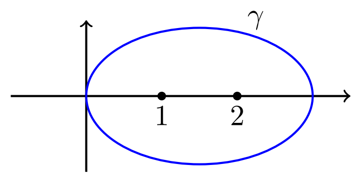
Math 444 - Homework 9 **Name:** ______________________

1. Let $\gamma(t) = 2e^{2it} - e^{it}$, $0 \le t \le 2\pi$. This path loops around the origin twice as shown below. Calculate $\displaystyle\int_\gamma \frac{dz}{z}$ for this path. Hint: You can make it easier if you break the path into two simple closed curves, an inner one and an outer one, then apply the Cauchy Integral Formula.

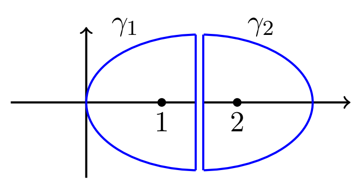


2. Let γ be the ellipse $|z-1| + |z-2| = 3$. Use a partial fraction decomposition (i.e., find the constants A and B below) to calculate

$$\oint_\gamma \frac{z}{(z-1)(z-2)}\,dz = \oint_\gamma \frac{A}{z-1} + \frac{B}{z-2}\,dz.$$



3. What if you calculate the integral in problem 2 by splitting the elliptical path into a sum of two separate integrals along positively oriented paths γ_1 and γ_2 as shown in the figure below? Find the values of $\displaystyle\oint_{\gamma_1} \frac{z}{(z-1)(z-2)}\,dz$ and $\displaystyle\oint_{\gamma_2} \frac{z}{(z-1)(z-2)}\,dz$. Check to see if the sum of these two integrals is the same as the integral in problem 2.



4. What is the power series for $f(z) = \dfrac{z}{z^2 - 2i}$ centered at $w = 0$? What is the radius of convergence for that power series?

Use Cauchy's integral formulas (including for derivatives) to evaluate the following.

5. $\displaystyle\oint_{|z-3|=2} \frac{e^z}{z(z-3)}\,dz$

6. $\displaystyle\oint_{|z|=4} \frac{e^z}{z(z-3)}\,dz$

7. $\displaystyle\oint_{|z|=4} \frac{\exp(3z)}{(z-\pi i)^2}\,dz$

8. $\displaystyle\oint_{|z|=3} \operatorname{Log}(z-4i)\,dz$

9. $\displaystyle\oint_{|z|=1} \frac{\cos(2z)}{z^3}\,dz$

10. $\displaystyle\oint_{|z|=3} \frac{\exp(2z)}{(z-1)^2(z-2)}\,dz$

11. Let $p(z) = (z-\frac{1}{2})(z-2)(z-\frac{i}{2})$. What is the winding number of the path $\gamma_1(t) = p(e^{it}), 0 \le t \le 2\pi$ around the origin? What about the path $\gamma_2(t) = p(3e^{it}), 0 \le t \le 2\pi$?

12. What is the winding number of the path $\gamma(t) = 2e^{3it} + 5e^{2it} - 3e^{it}$, $0 \le t \le 2\pi$ around the origin? Hint: $\gamma(t)$ is a polynomial function of e^{it}. What are the roots of that polynomial?