Infinite Series **Math 444 - Practice**

Find the sums of the following geometric series.

1. $\displaystyle\sum_{k=0}^{\infty} \frac{(3+4i)^k}{10^k}$

2. $1 + \frac{1}{2}i - \frac{1}{4} - \frac{1}{8}i + \frac{1}{16} + \frac{1}{32}i + \ldots$

A series $\sum_{k=0}^{\infty} a_k$ *converges absolutely* if $\sum_{k=0}^{\infty} |a_k|$ converges.

3. In this problem, we will prove that if $\sum_{k=0}^{\infty} a_k$ converges absolutely, then the sequence of partial sums $S_n = \sum_{k=0}^{\infty} a_k$ is a Cauchy sequence.

   (a) For any positive integers $n > m$, find an expression for $S_n - S_m$ in terms of the entries in the series a_k.

   (b) Use induction to prove the extended triangle inequality: $|z_1 + z_2 + \ldots + z_n| \leq |z_1| + |z_2| + \ldots + |z_n|$ for any positive integer n and any set of n complex numbers, $z_1, \ldots, z_n$.

   (c) Use the extended triangle inequality to explain why $|S_n - S_m| \leq \sum_{k=N}^{\infty} |a_k|$ when $n > m \geq N$.

   (d) Explain why the partial sums S_n are a Cauchy sequence.