Math 105 - Homework 6 **Name:** ________________________

Solve the following without using a calculator.

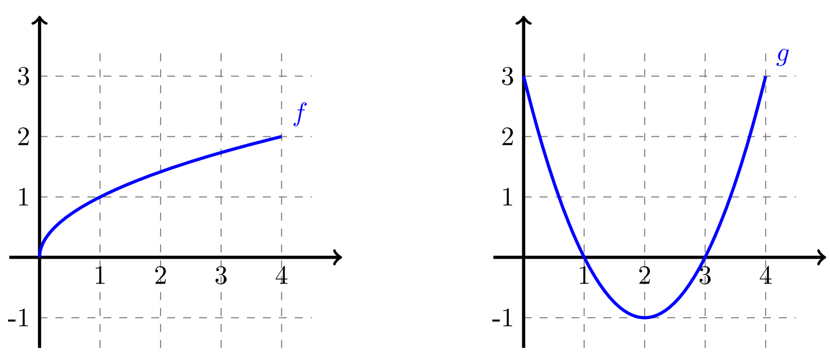
1. If $f(x) = 5 + x$ and $g(x) = \sqrt{x}$, then what are $f(g(4))$ and $g(f(4))$?

2. Suppose $f(x) = x^2 - 4$ and $g(x) = 2 - 3x$. How far apart are $f(3)$ and $g(3)$ on a number line?

3. The function $f(x) = \frac{1}{2}\left(x + \frac{5}{x}\right)$ can be used to calculate the square root of 5. Find $f(5)$ and $f(f(5))$. *Cool fact: if you kept going, every extra time you apply the function f to the previous answer, you would get closer and closer to $\sqrt{5}$ which is approximately* 2.236068.

4. Suppose that the population of a certain species is represented by the variable x. Let $f(x)$ be a function that predicts what the population will be one year later, based on the current population. What would the function $f(f(x))$ represent?

5. The following graphs show two different functions $f(x)$ and $g(x)$.



   Use the graphs to evaluate $g(f(4))$ and $f(g(1))$.

6. Sketch a graph of the function $f(x) = 4 - (x + 1)^2$ by plotting the y-values at $x = 0$, ± 1, and ± 2 and then filling in the rest of the graph.

7. The amount of garbage produced by a city (measured in tons per week) is given by a function $g(p)$ where the variable p is the city's population measured in thousands of people. One city has a population of 40,000 people and produces 13 tons of garbage each week. In function notation, this would be expressed as (fill in the blanks):

$$g(\qquad\qquad) =$$

8. The inverse of the function g in the last problem would be written g^{-1}. Explain what the information $g^{-1}(5) = 18$ would tell us about a city. That is, what is its population and garbage production?

9. Suppose that $f(x)$ is a linear function such that $3 = f(0)$ and $5 = f(1)$. Find the formula for $f(x)$.

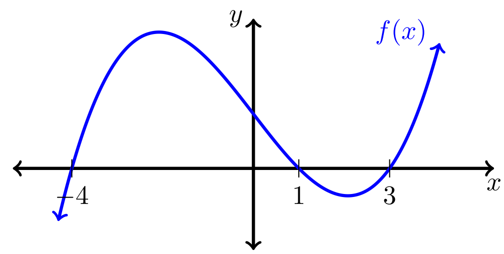
10. If $f(x) = 3x + 1$, then what is $f(f(x))$? Simplify your answer.

11. The time in seconds that it takes a pendulum to complete a full oscillation (swing back and forth) is $T = 2\pi\sqrt{\frac{L}{9.8}}$ where L is the length of the pendulum in meters. Find the inverse of this function.

12. The function $A(r) = \pi r^2$ computes the area of a circle of radius r. Find the formula for the inverse function and describe in words what it computes about a circle.

13. What is the domain of the function $h(x) = \sqrt{6 - x}$? That is, what x-values make sense as inputs?

14. Use the graph below to find the values of x for which $f(x) = x^3 - 13x + 12 > 0$.



15. A bakery sells cupcakes. If they gave away cupcakes for free, people would demand 1200 cupcakes per day. For every dollar the price of a cupcake increases above 0, they will sell 200 fewer cupcakes per day. Find a formula for the quantity of cupcakes $Q(p)$ they will sell as a function of the price p of a cupcake in dollars.

16. Find the total revenue $R(p)$ that the bakery in the previous problem will earn selling cupcakes as a function of p. Recall that revenue is price times quantity sold.

A store can produce souvenir T-shirts at a cost of \$2 each. They need to choose a price for the shirts. If they sell the shirts for \$5 each, they will sell $4,000$ shirts. If they raise the price, then for each \$1 increase in price, 400 fewer shirts will be sold. Using the variable p to represent the price that the store charges, find each of the following functions:

17. Quantity of shirts sold: $Q(p)$

18. Revenue (total money they get from selling shirts): $R(p)$

19. Cost (total money they spend to make the shirts): $C(p)$

20. Profit (revenue minus cost): $P(p)$