Homework 1 - Math 243 **Name:** ______________________

1. Compute the derivative of $y(t) = 7e^{t^2} + 3$ and use it to show that $y(t)$ is a solution to the differential equation

$$\frac{dy}{dt} = 2ty - 6t.$$

2. Which of the following functions is also a solution to the differential equation $y' = 2ty - 6t$?
   (a) $y(t) = Ae^{t^2} + 3$
   (b) $y(t) = 7e^{t^2} + B$
   (c) $y(t) = 7e^{t^2+C} + 3$

3. Substitute e^{at} for $y(t)$ in each of the following differential equations. Then simplify and find all values of the constant a such that $y = e^{at}$ is a solution.
   (a) $y'' + 2y' - 15y = 0.$

   (b) $y''' - 4y'' + 4y' = 0.$

4. A stockpile of nuclear waste initially contains 0.8 kilograms of radium. The radium decays exponentially at rate r, but new radium waste is added to the stockpile at a rate of 0.02 kilograms per year. Write an differential equation modeling the mass of radium in the stockpile. You don't need to look up the value of the constant r.

Solve the following separable equations with the given initial values.

5. $x\,dx - y^2\,dy = 0$, with $y(0) = 1$.

6. $\dfrac{dy}{dt} = \dfrac{\cos t}{y}$, with $y(0) = 5$.

7. $xy' = \sqrt{1 - y^2}$, with $y(1) = 0$.

   Hint: recall that $\dfrac{d}{dy} \arcsin(y) = \dfrac{1}{\sqrt{1 - y^2}}$.

8. $\dfrac{dy}{dt} = y^2$ with $y(0) = 2$.