Homework 10 - Math 243   Name: _____

Complexify the forcing term to find a particular solution for each of the following differential equations.

1. $y'' + 9y = \sin 4t$

2. $x'' + 2x' + 10x = e^t \cos t$
   Hint: $e^t \cos t$ is the real part of $e^{(1+i)t}$.

Find a particular solution for each of the following non-homogeneous linear equations.

3. $x'' + 7x' + 10x = e^{-5t}$

4. $y'' + 9y = \sin 3t$

5. If a linear differential equation
$$ay'' + by' + cy = f(t) + g(t)$$
has two different forcing functions f and g added together, then (by linearity) you can find a particular solution by adding together particular solutions when you only use f as the forcing function and when you only use g as the forcing function. Use this to find a particular solution to
$$y'' + 100y = e^t + \cos t.$$

6. The nonlinear system

$$x' = 5y - xy$$
$$y' = 2x - xy$$

has two equilibrium points $(0, 0)$ and $(5, 2)$. Calculate the Jacobian matrix at each equilibrium point and use it to determine whether the equilibrium is a source/sink, spiral source/sink, or a saddle.

7. Consider the nonlinear system

$$x' = y - x^2,$$
$$y' = y - 4.$$

Find all equilibrium points and classify each equilibrium as a saddle, source/sink, or spiral source/sink.