Homework 3 - Math 243 **Name:** ____________________

Solve the following initial value problems using separation of variables.

1. $\frac{dy}{dt} + y = 5$, $y(0) = 20$.

2. $\frac{dx}{dt} = 3t^2x + x$, $x(0) = 3$.

3. Consider the differential equation $\frac{dx}{dt} = x - t^2$ with initial value $x(0) = 3$. Without using a computer, compute the first 3 steps of Euler's method with a step size of $h = 1$. Enter your results in the table below.

t	x
0	3
1	
2	
3	

4. Consider the differential equation $\frac{dy}{dt} = 3y - 1$ with initial value $y(0) = 2$.

   (a) Use separation of variables to solve this differential equation.

   (b) Use Euler's method on a computer to estimate $y(2)$ using a step size of $h = 0.5$.

   (c) How far apart is the exact value of $y(2)$ and the Euler's method approximation (accurate to 4 decimal places)?

5. Suppose that a simple electric circuit has a resistor with resistance R in ohms (Ω), a capacitor with capacitance C in farads (F) and a (time-dependent) voltage source that provides $E(t)$ volts (V). The voltage drop across the capacitor E_C satisfies the differential equation

$$RC\frac{dE_C}{dt} + E_C = E(t).$$

Suppose that $R = 2\ \Omega$, $C = 1$ F, and $E(t) = 5\sin(2\pi t)$ volts where t is measured in seconds. If the initial voltage drop across the capacitor is $E_C(0) = 10$ V, then use Euler's method with a step size of $h = 0.1$ seconds to estimate the $E_C(t)$ when $t = 5$.

6. The velocity of an object falling near the surface of the Earth is governed by the drag equation:

$$m\frac{dv}{dt} = mg - \tfrac{1}{2}\rho v^2 A C_d$$

where m is the mass of the object, g is the acceleration of gravity, ρ is the density of the fluid in which the object is falling, A is the cross-sectional area of the object, and C_d is the coefficient of drag. What is the equilibrium solution to this equation? Is it stable? And what does it mean about the falling object?

7. Use Euler's method with $h = 0.01$ to estimate the velocity of a 1 kg sphere with a radius of 0.1 meters that has been falling for 10 seconds. Assume that $g = 9.8$ m/s^2, $C_d = 0.47$, $\rho = 1.2$ kg/m^3, and $A = 0.314$ m^2.