Homework 6 - Math 243 **Name:** _____

1. The **trace** of a matrix A (denote $\operatorname{tr} A$) is the sum of the entries on the main diagonal. For example,

$$\operatorname{tr}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = a + d.$$

(a) Show that the characteristic polynomial of any 2-by-2 matrix A is $x^2 - (\operatorname{tr} A)x + \det A$.

(b) If A is a 2-by-2 matrix with eigenvalues λ_1 and λ_2, show that the characteristic polynomial of A is also

$$x^2 - (\lambda_1 + \lambda_2)x + \lambda_1\lambda_2.$$

Hint: Since λ_1 and λ_2 are the roots, the characteristic polynomial factors as $(x - \lambda_1)(x - \lambda_2)$.

(c) What's the relationship between $\operatorname{tr} A$ and the eigenvalues of A? Hint: Compare the coefficients of the two formulas for the characteristic polynomial.

(d) What's the relationship between $\det A$ and the eigenvalues of A?

2. $\lambda = 4$ is an eigenvalue for the matrix $A = \begin{bmatrix} 1 & -2 \\ -3 & 2 \end{bmatrix}$. Find a corresponding eigenvector by finding the null space of $A - \lambda I$.

For each of the following matrix-vector pairs, show that the vector is an eigenvector and find its eigenvalue.

3. $A = \begin{bmatrix} -1 & 6 \\ -2 & 6 \end{bmatrix}$, $v = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

4. $B = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$, $w = \begin{bmatrix} 1 \\ -i \end{bmatrix}$

Find the characteristic polynomial and eigenvalues for each of the following matrices.

5. $\begin{bmatrix} 4 & 5 \\ 0 & 3 \end{bmatrix}$

6. $\begin{bmatrix} 1 & -4 \\ 4 & 9 \end{bmatrix}$

Use a computer to find the eigenvectors and eigenvalues for the following matrices. Be clear about which eigenvalue corresponds to which eigenvector.

7. $\begin{bmatrix} 5 & 1 & 1 \\ -3 & 1 & -3 \\ 0 & 0 & 4 \end{bmatrix}$

8. $\begin{bmatrix} 3 & 4 \\ -4 & 3 \end{bmatrix}$

Find general solutions for the following linear systems. You can use a computer to find the relevant eigenvectors/eigenvalues.

9. $\begin{aligned} x' &= 4x + 2y \\ y' &= 1x + 3y \end{aligned}$

10. $\begin{aligned} x' &= -3x + 4y \\ y' &= 3x - 2y \end{aligned}$

Find solutions for the following initial value problems. You should start with the general solutions from the last two problems.

11. $\begin{aligned} x' &= 4x + 2y \\ y' &= 1x + 3y \end{aligned}$, $x(0) = 2$ and $y(0) = 7$.

12. $\begin{aligned} x' &= -3x + 4y \\ y' &= 3x - 2y \end{aligned}$, $x(0) = 8$ and $y(0) = -6$.