Homework 8 - Math 243

Name:

Each of the following linear systems has a single parameter a. Calculate the trace and determinant for each system, and use them to find the values of a where the type of equilibrium changes.

1.
$$\frac{d\mathbf{x}}{dt} = \begin{bmatrix} a & -1\\ 2 & 0 \end{bmatrix} \mathbf{x}.$$

$$2. \ \frac{d\mathbf{y}}{dt} = \begin{bmatrix} a & a \\ 1 & 0 \end{bmatrix} \mathbf{y}.$$

- 3. Consider the 2-parameter family $\frac{d\mathbf{y}}{dt} = \begin{bmatrix} 1 & a \\ -b & 0 \end{bmatrix} \mathbf{y}$. Describe in words how the type of equilibrium depends on the parameters a and b.
- 4. If a 2-by-2 matrix A has one eigenvalue equal to 4, express the trace and determinant of A as functions of the other eigenvalue λ . What types of equilibria are possible for the system $\mathbf{x}' = A\mathbf{x}$ with different values of λ ?

Use the matrix exponential function (on a computer) to solve the following initial value problems.

$$x' = x + 2y$$

$$y' = 3x - 4y$$

$$x(0) = 1$$

$$y(0) = 0.$$

$$x' = 2x + y$$

$$6. \quad y' = 2y$$

$$x(0) = 1$$

$$y(0) = 2.$$

$$x' = 3x - y$$

$$y' = -x + 3y$$

$$z' = 5x - 5y - 6z$$

$$x(0) = 4$$

$$y(0) = 2$$

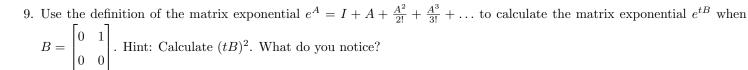
$$z(0) = -3$$

$$x' = -x + 6y$$

$$y' = -6x - y$$

$$x(0) = 3$$

$$y(0) = 0.$$



10. The matrix
$$A = \begin{bmatrix} 2 & -3 \\ 0 & -1 \end{bmatrix}$$
 has eigenvectors $v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and corresponding eigenvalues $\lambda_1 = 2$ and $\lambda_2 = -1$.

(a) Use the formula
$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$
 to find the inverse of $V = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.

(b) Calculate $D = V^{-1}AV$ by hand and show that it is a diagonal matrix.

(c) Calculate
$$e^{tA} = Ve^{tD}V^{-1}$$
 by hand. Recall that for a diagonal matrix $D = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$, $e^{tD} = \begin{bmatrix} e^{\lambda_1 t} & 0 \\ 0 & e^{\lambda_2 t} \end{bmatrix}$.