Find the equilibrium point for each of the following non-homogeneous linear systems.

1. $\dfrac{d\mathbf{x}}{dt} = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 6 \\ -2 \end{bmatrix}.$

2. $\dfrac{d\mathbf{y}}{dt} = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix} \mathbf{y} + \begin{bmatrix} 2 \\ 3 \end{bmatrix}.$

Find the general solutions for each of the following non-homogeneous linear systems.

3. $\dfrac{d\mathbf{x}}{dt} = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 6 \\ -2 \end{bmatrix}.$

4. $\dfrac{d\mathbf{y}}{dt} = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix} \mathbf{y} + \begin{bmatrix} 2 \\ 3 \end{bmatrix}.$

Find particular solutions for each of the following differential equations.

5. $x'' - 2x' - 3x = 6e^{2t}$

6. $y'' - y' - 2y = 4t^2$

7. $y'' - 4y = \cos t$

8. $z'' - 4z = \cos 2t$

Solve the following initial value problems.

9. $x'' - 2x' - 3x = 6e^{2t}$ with $x(0) = 2$, $x'(0) = 0$.

10. $y'' - 4y = \cos t$ with $y(0) = 4$, $y'(0) = 3$.

11. Find the general solution for the following non-homogeneous linear system.

$$\frac{dX}{dt} = \begin{bmatrix} 1 & -1 \\ 5 & -1 \end{bmatrix} X + \begin{bmatrix} 4t \\ 8t \end{bmatrix}.$$

Hint: The particular solution has the form $\mathbf{a}t + \mathbf{b}$ where $\mathbf{a}$ and $\mathbf{b}$ are constant vectors.