The **Laplace transform** of a function $f(t)$ is

$$F(s) = \mathcal{L}(f)(s) = \int_0^\infty e^{-st} f(t)\, dt.$$

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1. Calculate the Laplace transform of a constant function $f(t) = c$.

2. The most important property of the Laplace transform is how it interacts with derivatives. Use integration by parts to prove that

$$\mathcal{L}\left(\frac{d}{dt} f(t)\right) = s \cdot \mathcal{L}(f) - f(0).$$

3. Find the Laplace transforms of $\cos t$ and $\sin t$. Hint: Complexify. The Laplace transforms of $\cos t$ and $\sin t$ are the real and imaginary parts of the Laplace transform of e^{it}.

4. How could you find $\mathcal{L}(t^n)$ for any positive integer n?