
The **characteristic polynomial** of an n-by-n matrix A is

$$\det(A - \lambda I)$$

where I is the **identity matrix**. Recall that the determinant of a 2-by-2 matrix is

$$\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc.$$

The roots of the characteristic polynomial are the **eigenvalues** of A. If λ is an eigenvalue of A, then any non-zero vector x such that $Ax = \lambda x$ is called an **eigenvector**. You can find the eigenvectors by finding the null space of the matrix $A - \lambda I$.

---

1. Find the characteristic polynomials and eigenvalues for the following matrices
   (a) $\begin{bmatrix} 5 & -6 \\ 3 & -4 \end{bmatrix}$.
   (b) $\begin{bmatrix} 2 & -3 \\ 3 & 2 \end{bmatrix}$.
   (c) $\begin{bmatrix} 1 & -4 \\ 0 & 3 \end{bmatrix}$.

2. Show that $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ is an eigenvector of $\begin{bmatrix} 7 & -10 \\ 5 & -8 \end{bmatrix}$. What is the corresponding eigenvalue?

3. Find the eigenvectors of $A = \begin{bmatrix} 1 & -4 \\ 0 & 3 \end{bmatrix}$ by hand.

4. Finding the eigenvectors of matrices can be tedious, so we will often use computers to do it quickly. Use a computer to find the eigenvectors of the following matrices.
   (a) $\begin{bmatrix} 5 & -6 \\ 3 & -4 \end{bmatrix}$.
   (b) $\begin{bmatrix} 2 & -3 \\ 3 & 2 \end{bmatrix}$.