Formula Sheet

Integrating Factors

A first order linear differential equation $\dfrac{dy}{dt} + f(t)y = g(t)$ has general solution

$$y(t) = \frac{\int e^{F(t)} g(t)\, dt}{e^{F(t)}}$$

where $F(t)$ is any antiderivative of $f(t)$.

Trace-Determinant Formula for Eigenvalues

For a 2-by-2 matrix A,

$$\lambda = \frac{\operatorname{tr} A \pm \sqrt{(\operatorname{tr} A)^2 - 4 \det A}}{2}.$$

Linear Systems

For a linear system $\dfrac{d\mathbf{x}}{dt} = A\mathbf{x}$:

Straight-line solutions $\mathbf{x}(t) = Ce^{\lambda t}\mathbf{v}.$	**Matrix exponential solution** $\mathbf{x}(t) = e^{At}\mathbf{x}(0).$
Complex eigenvalues If $\lambda = \alpha \pm i\beta$ is a complex eigenvalue with eigenvector $\mathbf{v}$, then the real and imaginary parts of $e^{\alpha t}(\cos(\beta t) \pm i \sin(\beta t))\mathbf{v}$ are both real-valued solutions.	**Repeated eigenvalues** If A is a 2-by-2 matrix with a repeated eigenvalue λ, then the solution is $\mathbf{x}(t) = e^{\lambda t}(I + t(A - \lambda I))\mathbf{x}(0).$