Markov Chains with Rewards Workshop **CS 480**

Write your answers below. When you are finished, turn this page in, but also share your Python code with me. If you are using Google Colab, share your code with: `lins.brian@gmail.com`

1. **Gambler's ruin problem.** Two gamblers are playing a game. Player 1 wins with probability p and player 2 wins the rest of the time (so with probability $1 - p$). Each player starts with n dollars, and whoever wins the game takes \$1 from the loser. They keep playing repeatedly until one player runs out of money.

   (a) Write a function that constructs the transition matrix for this Markov chain. Your function should accept n and p as inputs and return a numpy array. Hint: There are $2n + 1$ states (which can be labeled with the amount of money player 1 has).

   (b) Write a function to perform value iteration for any transition matrix Q and reward vector R. It should return a value vector v as a numpy array.

   (c) Use your value iteration function to calculate the average length of a game if both players start with \$5 and player 1 wins each round 60% of the time. Hint: You can use a reward vector with $R_i = 1$ for each state i, except the two end states.

   (d) **Raising the stakes.** What if each player starts with \$100? How long will the game last (on average)?

   (e) **I'll drink to that.** Suppose that player 2 also has a side bet where he earns one free drink for every round he has more money than player 1. How many drinks will player 2 earn on average? Assume that each player starts with \$5 and player 1 has a 60% chance to win each round.

2. **Coupon collector's problem.** A store has n different kinds of special coupons. A collector wants to collect at least one of each type of coupon. Every time the collector goes to the store, he gets one coupon which is equally likely to be any of the n types.

   (a) Write a function that constructs the transition matrix for this Markov chain.

   (b) How long, on average, will it take the coupon collector to collect all n coupons if $n = 20$?