

## Hamilton's Method

## Math 111 - Workshop

*Hamilton's method has certain surprising paradoxes that can happen. Here we will look at two of these: the population paradox, and the new states paradox.*

1. The Metro Garbage Company provides service to the two districts of Metropolis: Northtown (with 10,450 homes) and Southtown (with 89,550 homes).

District	Northtown	Southtown	Total
Homes	10,450	89,550	100,000

- (a) If the company runs 100 garbage trucks that it apportions to the districts according to Hamilton's method, how many trucks will each district receive? *Hint: in this part it is very easy to find the standard divisor, you don't even need a calculator, so use the standard divisor to find the standard quotas.*
- (b) Suppose that the Metro Garbage Company adds the 5,250 homes in Newtown to its list of customers. To cover the additional homes, the company buys 5 new trucks which is exactly the number of trucks that will be needed to cover Newton. Check that this is correct by finding the new apportionment using Hamilton's method.

District	Northtown	Southtown	Newtown	Total
Homes	10,450	89,550	5,250	105,250

- (c) What happened to the apportionment of trucks between Northtown and Southtown? Which district lost a truck?
- (d) Which paradox is this?

2. At Collegeville University, there are 4 divisions, (Arts, Business, Humanities, and Science). The school wants to apportion its 250 faculty as fairly as possible between the four divisions.

School	Arts	Business	Humanities	Science	total
Students	884	1675	1225	216	4000

- (a) What is the student-teacher ratio at Collegeville University?
- (b) In apportionment problems, the student-teacher ratio is the same as which number? Is it the standard quota, population, standard divisor? Explain.
- (c) Using Hamilton's method, what is the apportionment of the faculty to the 4 divisions.
- (d) Suppose that enrollment in the Arts division increases by 6 students, Business enrollment increases by 11, and Humanities enrollment increases by 3, but the Sciences stay flat. Use the new enrollment data shown below to find the new apportionment under Hamilton's method.

School	Arts	Business	Humanities	Science	total
Students	890	1686	1228	216	4020

- (e) Which division lost a professor, and which division gained a professor as a result of the enrollment change? Why is this result paradoxical?