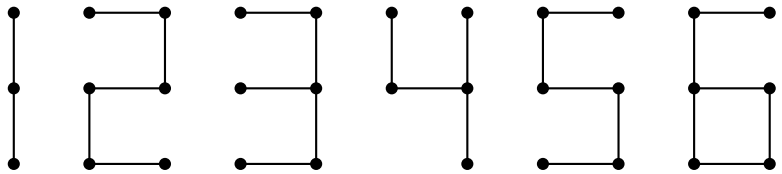
Trees **Math 111 - Workshop**

1. Which of the following graphs is a tree?



2. Two graphs are **isomorphic** if you can stretch and bend the edges so that they have the same shape. For example, the graphs

are isomorphic. Draw two different trees with four vertices that are not isomorphic.

3. Draw three different trees with five vertices that are not isomorphic.

4. For each of the following situations, determine whether the graph is (i) definitely a tree, (ii) definitely not a tree, or (iii) not enough information to tell. Explain your reasoning for each.

   (a) A connected graph has 10 vertices and 9 edges.

   (b) A connected graph has 4 vertices and they are all degree 3.

   (c) A connected graph with 8 vertices, and every vertex has degree 1 or degree 3.

5. A graph is called *regular* if every vertex has the same degree.
   (a) Draw a picture of a regular tree.

   (b) Explain why a tree with 3 or more vertices cannot be regular.

6. Draw a picture of a tree with a degree 5 vertex.

7. Explain why a tree that has a degree n vertex must have at least n degree 1 vertices, no matter how big n is.

8. Use Kruskal's algorithm to find a minimum spanning tree that connects the following cities. The distances between the cities in miles is shown below.

	Richmond	Washington, D.C.	Roanoke	Virginia Beach	Charlottesville
Richmond	0	108	189	107	71
Washington, D.C.	108	0	239	208	117
Roanoke	189	239	0	288	120
Virginia Beach	107	208	288	0	154
Charlottesville	71	117	120	154	0