Math 141 - Homework 7

Name: _____

Find the derivative of each function.

1. $f(x) = \sqrt{\dfrac{x-1}{x+1}}$

2. $y = x \sin \dfrac{1}{x}$

3. $y = \sqrt{1 + \sqrt{1+x}}$

4. $\sin(\sin(\sin x))$

5. Suppose that the average price of a house in one region is currently $h(t) = \$400,000$ and going up at a rate of $\$20,000$ per year.

   (a) From the description above, what is $\dfrac{dh}{dt}$ and what are its units?

   (b) An economist estimates that the population of homeless people will increase by about 0.1 for every dollar higher average home prices get. In other words, $\dfrac{dP}{dh} = 0.1$. Use this to estimate the current rate of change in the homeless population per year.

Use implicit differentiation to find $\dfrac{dy}{dx}$ for each of the following equations.

6. $y^2 - x^2 = 9$

7. $x^2 y = y - 5$

8. $xy - \sin(xy) = 1$

9. $\sqrt{xy} = 1 + y^2$

10. $\tan(y) = x$

11. $\dfrac{1}{y} = x + \dfrac{1}{x}$

Use implicit differentiation to find the equation of the tangent line to the curve at the indicated point. Then draw and label a graph showing both the curve and the tangent line. Hint: You can use Desmos to help.

12. (Ellipse) $x^2 + 2xy + 3y^2 = 6$ at $(1, 1)$

13. (Tschirnhausen Cubic) $y^2 = x^3 + 3x^2$ at $(1, 2)$.

14. Find all points on the ellipse $x^2 - xy + y^2 = 3$ where the tangent line is horizontal.