Calculus I - Math 141 **Final Exam**

Name: ___________________________

You must show all work to earn full credit. No calculators allowed. If you do not have room in the given space to answer a question, use the back of another page and indicate clearly which work goes with which problem.

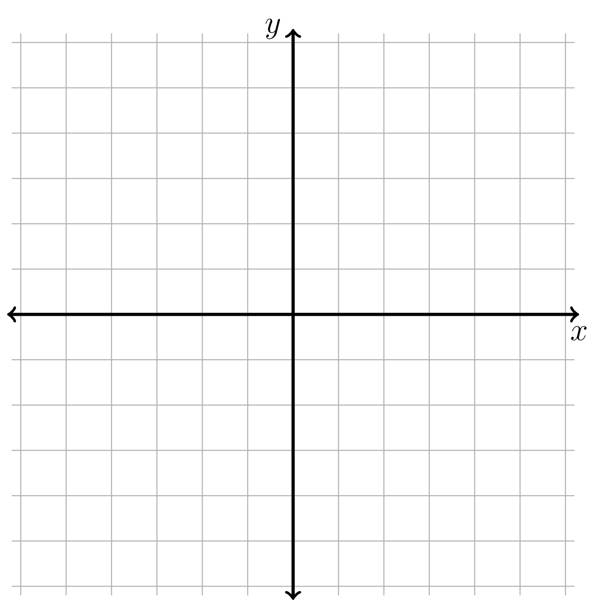
Problem	Maximum Points	Your Score
1	8	
2	9	
3	8	
4	9	
5	8	
6	6	
7	8	
8	8	
9	6	
10	6	
11	6	
12	4	
13	8	
14	6	
Total:	100	

1. (8 points) Let $f(x) = x^3 - 3x^2 + 4$.

(a) Find the intervals of increase/decrease for $f(x)$.

(b) Find the intervals of concavity.

(c) Sketch a graph of the function using the axis provided below.



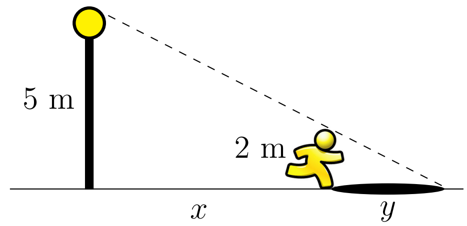
2. (9 points) Find the following derivatives.

(a) $\dfrac{d}{dx}\dfrac{x^2+\sqrt{x}}{x}$

(b) $\dfrac{d}{d\theta}\cos(\theta^2+2)$

(c) $\dfrac{d}{dx}x^2\sin 2x$

3. (8 points) A jogger, who is 2 meters tall, passes under a 5 meter street lamp running at 3 m/s.



(a) Use similar triangles to find a formula relating x and y.

(b) How fast is the jogger's shadow growing when the jogger has gone 6 meters past the lamppost?

4. (9 points) Find the following antiderivatives.

(a) $\displaystyle\int 6x^2 + 2x + 1\,dx.$

(b) $\displaystyle\int \sqrt{x}\,dx.$

(c) $\displaystyle\int 2x\cos(x^2)\,dx.$

5. (8 points) Let $f(x) = x(6 - x)$.

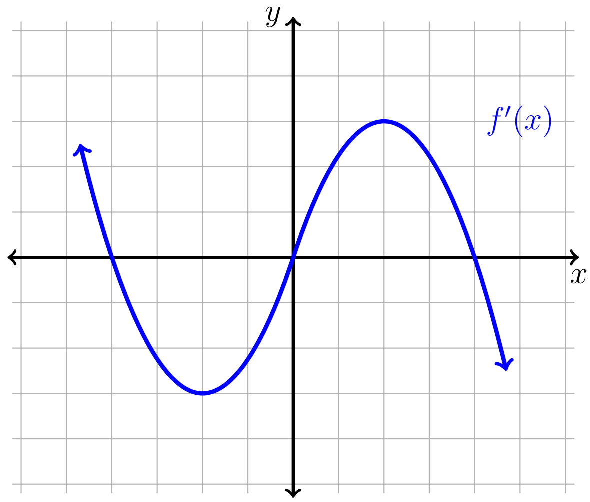
(a) Find the area under the curve $y = x(6 - x)$ from $x = 0$ to $x = 3$.

(b) Find average value of $f(x) = x(6 - x)$ on the interval $[0, 3]$.

6. (6 points) Find the value of c that makes the following function continuous.

$$f(x) = \begin{cases} cx - 2 & \text{if } x > 3 \\ 1 - x^2 & \text{if } x \leq 3 \end{cases}$$

7. (8 points) The graph below shows the **derivative** $f'(x)$ of another function $f(x)$.

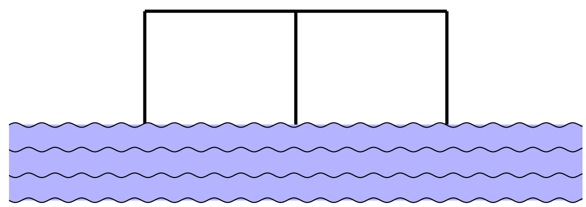


(a) Find the intervals where the original function $f(x)$ (which is not shown) is increasing.

(b) Find the x-values of all local maxes and local mins for $f(x)$.

(c) Use the graph of $f'(x)$ to find the intervals of concavity for $f(x)$.

8. (8 points) A farmer has 2400 ft. of fence and wants to fence off a rectangular field that borders a straight river. He also wants to use fence to divide the field into two halves as shown below. No fence is needed along the side of the river.



(a) Write down a formula for the area of the field. If you use variables, clearly indicate what each letter represents.

(b) Find the dimensions of the field with maximum possible area given the amount of fence available.

9. (6 points) Use linear approximation or differentials to approximate $\sqrt[3]{8.24}$.

10. (6 points) Use the *definition of derivative* to find the derivative of $f(x) = x^2 + 4$.

11. (6 points) A particle moves along the x-axis. The velocity of a particle is given by the formula $v(t) = 16t - 10$.

    (a) What is the acceleration of the particle as a function of time?

    (b) What is the position of the particle as a function of time, if the particle is at $x = 5$ when $t = 1$?

12. (4 points) If $h'(t)$ represents the rate of growth of a tree in feet per year, then what does

$$\int_3^5 h'(t)\,dt$$

represent about the tree?

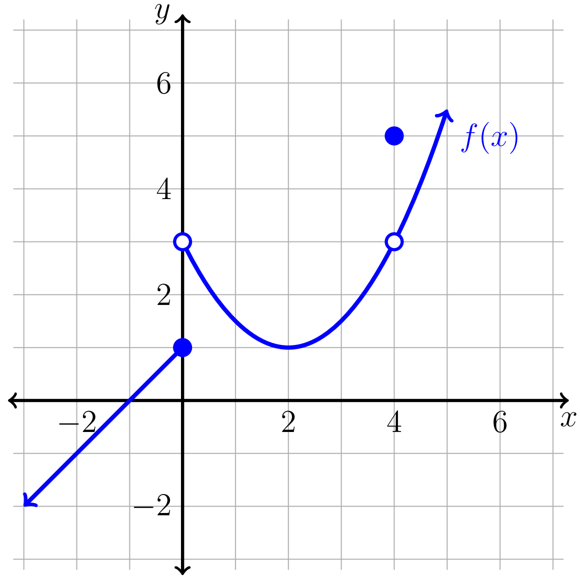
13. (8 points) Consider the function $f(x) = \dfrac{2x^2+6}{x^2+2x-3}$.

(a) Does $f(x)$ have any vertical asymptotes? If so, where?

(b) Does $f(x)$ have any horizontal asymptotes? Where?

(c) Is $f(x)$ a continuous function everywhere? Why or why not?

14. (6 points) Use the graph below to find the indicated limits.



(a) $\lim_{x \to 0} f(x)$.

(b) $\lim_{x \to 4} f(x)$.

Formula Sheet

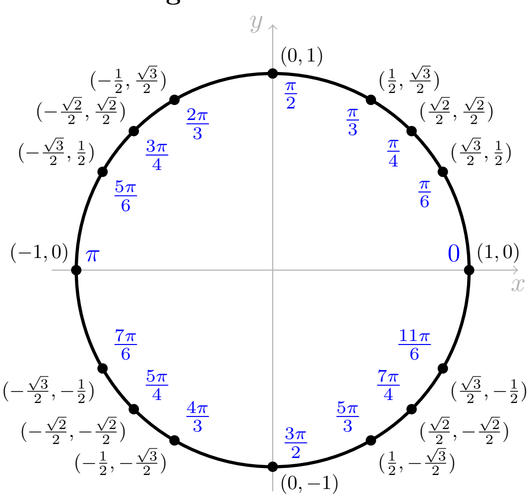
Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Point-Slope Form

$$y - y_1 = m(x - x_1)$$

Common Angles



Trigonometry Ratios

- $\tan x = \dfrac{\sin x}{\cos x} \qquad \cot x = \dfrac{\cos x}{\sin x}$
- $\sec x = \dfrac{1}{\cos x} \qquad \csc x = \dfrac{1}{\sin x}$

Angle Addition Identities

- $\cos(a + b) = \cos a \cos b - \sin a \sin b$
- $\sin(a + b) = \sin a \cos b + \sin b \cos a$

Trigonometry Limits

- $\displaystyle\lim_{x \to 0} \frac{\sin x}{x} = 1$
- $\displaystyle\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$

Definition of Derivative

- $\displaystyle f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$, or
- $\displaystyle f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

Selected Derivatives

- $\dfrac{d}{dx} \tan x = \sec^2 x$
- $\dfrac{d}{dx} \sec x = \sec x \tan x$
- $\dfrac{d}{dx} \cot x = -\csc^2 x$
- $\dfrac{d}{dx} \csc x = -\csc x \cot x$

Linear Approximation

- $f(x) \approx f(a) + f'(a)(x - a)$

Error and Relative Error

- $dy \approx$ the error in y
- $\dfrac{dy}{y} \approx$ the relative (percent) error in y

Newton's Method

- $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

Summation Formulas

- $\displaystyle\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$
- $\displaystyle\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

Riemann Sum

- $\displaystyle A \approx \sum_{n=1}^{N} f(x_n)\Delta x$ where
- $\Delta x = \dfrac{b - a}{N}$ and
- $x_n = a + n\Delta x$

Average Value of a Function

- $\displaystyle \frac{1}{b - a} \int_a^b f(x)\, dx$