Numerical Integration **Math 342**

In this workshop, we will compare two different integration techniques: integrating Taylor series versus composite Newton-Cotes methods (like the trapezoid rule and Simpson's method).

1. Let $f(x) = \dfrac{\sin x}{x}$. Find the Maclaurin series for $f(x)$.

2. The **sine integral** $\mathrm{Si}(x)$ is a special function defined by the formula
$$\mathrm{Si}(x) = \int_0^x \frac{\sin t}{t}\, dt.$$
Integrate the Maclaurin series for $\dfrac{\sin t}{t}$ to find the Maclaurin series for $\mathrm{Si}(x)$.

3. Write a program to compute $\mathrm{Si}(\pi)$ by adding up the first 10 terms of the Maclaurin series for $\mathrm{Si}(x)$. What do you get?

4. Compute $\mathrm{Si}(\pi)$ using the trapezoid rule with $n = 100$ trapezoids. What do you get?

5. According to WolframAlpha,
$$\mathrm{Si}(\pi) = 1.851937051982466170361053370157991363345809728981154909804783781 8\ldots$$
Which method is more accurate, the Trapezoid rule with $n = 100$ or the Maclaurin series with 10 terms?

Normal Distribution. The standard normal distribution in statistics has probability density function

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}.$$

When something that is randomly distributed with the normal distribution happens, the probability that the outcome has an x-value that is between a and b is the integral

$$\int_a^b \frac{1}{\sqrt{2\pi}} e^{-x^2/2}.$$

Unfortunately, this function does not have a nice antiderivative.

6. Find the Maclaurin series for $e^{-x^2/2}$.

7. Integrate the Maclaurin series for $e^{-x^2/2}$ to find the normal distribution cumulative density function

$$\Phi(x) = \tfrac{1}{2} + \frac{1}{\sqrt{2\pi}} \int_0^x e^{-t^2/2}\, dt.$$

8. Compute $\Phi(2)$ by using the first 10 terms of the Maclaurin series you found above.

9. Compute $\Phi(2)$ by using the trapezoid rule with $n = 100$ trapezoids.