Numerical Differentiation **Math 342**

The definition of the derivative is $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$. The fraction on the right is called the **difference quotient**. If you choose h small, you can approximate the derivative by using the difference quotient.

1. Use a computer or calculator to evaluate the difference quotient for $f(x) = x^2$ at the point $x = 1$ with $h = 0.001$. What do you get?

2. Using the answer to the previous problem as an approximation for the derivative of $f(x) = x^2$ at $x = 1$, what is the relative error in the approximation?

3. Find a formula for the relative error when you use a difference quotient with $h = 10^{-k}$ to approximate the derivative of $f(x) = x^2$ at $x = 1$, as a function of k.

4. Use Desmos or Python to graph the natural log of the relative error as a function of k. What k appears to have the smallest relative error (roughly)?

5. Compute and graph the natural logarithm of the relative error when you use the difference quotient to approximate the derivative of $\sin x$ at $x = \frac{\pi}{3}$ with $h = 10^{-k}$, as a function of k.

6. The **centered difference quotient** $f'(x) \approx \dfrac{f(x+h) - f(x-h)}{2h}$ is a more accurate approximation of the derivative. Find the natural log of the relative error with this approximation for $f(x) = \sin x$ at $x = \frac{\pi}{3}$ with $h = 10^{-k}$, as a function of k. Compare the relative errors of the difference quotient versus the centered difference quotient. What k (roughly) minimizes the relative error for the centered difference quotient? How much more accurate is it than the best results you get with the regular difference quotient?