Interpolation **Math 342**

Consider the points $(0,0)\,,(1,1)\,,(2,-2)\,,(3,3)$.

1. Find a formula for a 3rd degree interpolating polynomial p_3 using the Lagrange basis for the points above.

2. Use divided differences to find the formula for p_3 in the Newton basis.

3. What is the Vandermonde system for these four points? You don't need to solve the system.

4. **Hermite Polynomials.** It is possible to create a polynomial for a function $f(x)$ where the polynomial not only has the same y-values at certain points, but also the slopes. The key is to repeat the x-values where you want the derivatives to match in the list of nodes. Then you can use the method of divided differences to find the Hermite interpolating polynomial (also called an osculating polynomial). Everything works almost exactly the same as the method for finding the regular interpolating polynomial in Newton form. The *only change* is that the divided difference for a repeated node x_i is

$$f[\underbrace{x_i, x_i, \dots, x_i}_{\text{repeated } k \text{ times}}] = \frac{f^{(k-1)}(x_i)}{(k-1)!}.$$

Use this idea to find the Hermite interpolating polynomial for the function $f(x) = \cos x$ by completing the following table of divided differences.

x	$f(x)$	DD1
$x_0 = -\pi$	-1	
		$f[x_0, x_0] = f'(-\pi) =$
$x_0 = -\pi$	-1	
		$f[x_0, x_1] = \frac{f(x_1)-f(x_0)}{x_1-x_0} =$
$x_1 = 0$	1	
		$f[x_1, x_1] = f'(0) =$
$x_1 = 0$	1	
		$f[x_1, x_2] = \frac{f(x_2)-f(x_1)}{x_2-x_1} =$
$x_2 = \pi$	-1	
		$f[x_2, x_2] = f'(\pi) =$
$x_2 = \pi$	-1	