Numerical Analysis - Math 342 **Midterm 1 Review**

The following problems are similar to ones you might see on the midterm exam.

1. Use Newton's method to write down an iterative formula for finding the root of $f(x) = x^3 - a$ for any constant a. If you start with the initial guess $x_0 = \frac{1}{3}a$, then what is x_1?

2. The root of $x^3 - 2$ is $\sqrt[3]{2}$, which is located in the interval $[1, 2]$. If we use the bisection method to find this root, starting with the endpoints $a = 1$ and $b = 2$, then what is the worst case error in our estimate for the root after 10 steps?

3. Find values for the constants M and L such that $|f''(x)| \le M$ and $|f'(x)| \ge L$ when $f(x) = x^3 - 2$ on the interval $[1, 2]$.

4. Based on your constants from the previous problem, and the Newton's method error formula

$$|x_{n+1} - r| \le \left(\frac{M}{2L}\right) |x_n - r|^2,$$

how close to the root r would the initial guess x_0 need to be in order to guarantee that Newton's method will converge?

5. Find the fixed points of the function $f(x) = \dfrac{8}{3x - 2}$.

6. What is the derivative of the function $f(x) = \dfrac{8}{3x - 2}$ at each fixed point? Based on the derivative, determine whether each fixed point is attracting or repelling (or not enough information).

7. Let $A = \begin{pmatrix} 1 & 2 & 4 \\ 5 & 7 & 21 \\ 1 & 11 & 1 \end{pmatrix}$.
   (a) Find the LU-decomposition of A.
   (b) What is the rank of A? Is A invertible?
   (c) Compute $\|A\|_\infty$.
   (d) Use the LU-decomposition to solve $Ax = \begin{pmatrix} 2 \\ 11 \\ -1 \end{pmatrix}$.

8. Suppose that $x = 1.234 \times 10^{-3}$ and $y = 1.225 \times 10^{-3}$ each have four significant digits. How many significant digits are there in each of the following numbers?
   (a) $x + y$.
   (b) $x - y$.
   (c) xy.
   (d) x/y.

9. Let $f(x) = \dfrac{e^x - 1}{x}$.
   (a) Find a Maclaurin polynomial for f by replacing e^x by its 3rd degree Maclaurin polynomial.
   (b) Find a formula for the error in the previous approximation using the Taylor remainder formula. What is an upper bound for the error on $[-1, 1]$?

10. If you use the secant method to find the root of $y = 2^x - 5$ starting with $x_0 = 1$ and $x_1 = 2$, what is x_2?