Homework 1 - Computer Science 461   **Name:** _____

Due Wed, Jan 22. You can send the code for the programming problem to `blins@hsc.edu`*.*

1. Write a logical expression using the operators $\wedge$ (AND), $\vee$ (OR), and $\neg$ (NOT) and the Boolean variables x_0, x_1, x_2 so that the expression is TRUE when an even number of the variables are TRUE and FALSE otherwise.

2. Translate the following mathematical short-hand into English:

$$\forall a, b \in \mathbb{Z} \text{ with } b \neq 0, \exists N \in \mathbb{N} : N > a/b \wedge N > b/a.$$

3. Describe the following sets in words. Just give a brief description of what objects are in the sets, you do not need to list all of the elements. (Note that $[n]$ denotes the first n natural numbers, so for example, $[4] = \{0, 1, 2, 3\}$).

   (a) $[9] \times [9] \times [9]$.

   (b) $\{x \in \{0, 1\}^{2n} : x_i = x_{i+n} \ \forall i \in [n]\}$.

4. Write a computer program to verify that $n^3 + (n + 1)^3 + (n + 2)^3$ is divisible by 9 for every integer $0 \leq n < 100$.

5. Use mathematical induction to prove that $n^3 + (n + 1)^3 + (n + 2)^3$ is divisible by 9 for every integer $n \geq 0$.