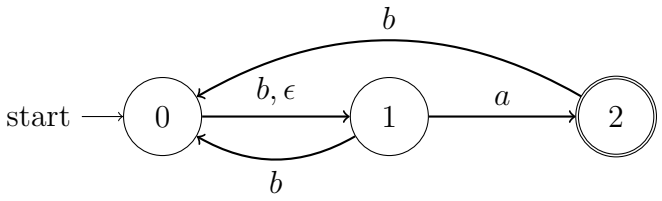
Homework 4 - Computer Science 461 **Name:** _____

Due Monday, Feb 10.

1. Convert the following NFA to a DFA. Use the method we discussed in class, where the states of the DFA correspond to subsets of the states of the original NFA. Hint: *After removing states in the DFA that you can never reach, you should only need a small number of states, one of which corresponds to the empty set.*



2. Let $\Sigma = \{0, 1\}$. Write a one-sentence description the languages defined by the following regular expressions. For example: Σ^*1 would be any binary string that ends with a 1.

   (a) $(\Sigma\Sigma)^*$.

   (b) $\Sigma^*01\Sigma^*$.

   (c) $(0\Sigma^*0)|(1\Sigma^*1)$.

   (d) $(00|01|11)^*$.

3. Let Σ be the regular English alphabet $\{a, b, c, \ldots, z\}$. Write a regular expression that matches all strings that contain at least two vowels (i.e., a,e,i,o,u).

4. Find a regular expression that matches each of the following languages. In all cases, the alphabet is $\Sigma = \{0, 1\}$.

   (a) $\{w \in \Sigma^* : w$ contains at least three 1's.$\}$

   (b) $\{w \in \Sigma^* : w$ contains at least two 1's and exactly one 0.$\}$

5. Prove that if $L \subset \Sigma^*$ is a regular language, then the complement $\Sigma^* \backslash L$ is also a regular language. Hint: *If there is a DFA $M = (Q, \Sigma, \delta, q, F)$ that recognizes L, describe a different DFA that recognizes the complement.*

6. Let $A \subseteq \Sigma^*$ be a regular language. Define a new language REMOVE-ONE(A) to be the set

$$\{x : xyz \in A \text{ where } y \in \Sigma, z \in \Sigma^*\}.$$

   In other words, REMOVE-ONE(A) is the set of all strings that can be obtained from strings in A by removing one letter. Prove that REMOVE-ONE(A) is a regular language by describing an NFA that recognizes it. Hint: Create an NFA using two copies of a DFA that recognizes A. Describe exactly how the NFA is constructed from the two DFAs.