Homework 9 - Computer Science 461 **Name:** ______________________

Due Monday, April 7.

1. Prove that if $A, B \subseteq \Sigma^*$ are both Turing decidable languages, then the intersection $A \cap B$ is also a decidable language.

2. Let $D \subset \Sigma^*$ be a decidable language. Prove that

$$C = \{x \in \Sigma^* : \text{ there exists } y \in \Sigma^* \text{ such that } xy \in D\}$$

is recognizable. Hint: Given a string $x \in \Sigma^*$, describe an algorithm you could implement using a Turing machine that decides D to determine if $x \in C$.

3. Use Rice's theorem to prove that the property EMPTY $= \{\langle M \rangle : L(M) = \varnothing\}$ is undecidable. That is, prove that there is no algorithm to decide whether a Turing machine accepts no strings.

4. A subset $S \subset \mathbb{N}$ is *decidable* if there is a computable function $f : \mathbb{N} \to \{0, 1\}$ such that $f(n) = 1$ if and only if $n \in S$. Give an informal argument to explain the following fact: A subset $S \subset \mathbb{N}$ is decidable if and only if there is a computer program that prints the elements of S *in increasing order.* Hint: Since the fact is an if-and-only-if statement, you'll have to explain both directions.

5. Let L be a Turing recognizable language that consists of binary descriptions of Turing machines

$$L = \{\langle D_0\rangle, \langle D_1\rangle, \langle D_2\rangle, \ldots\},$$

where every D_i is a decider (assume that every D_i has input alphabet $\Sigma = \{0, 1\}$). Prove that there is a decidable language in $\{0, 1\}^*$ that is not decided by any of the deciders D_i, $i \in \mathbb{N}$. Hint: Use a diagonalization argument on the strings in $\{0, 1\}^*$ to construct a new Turing machine N which decides a language $L(N)$ that is different from any of the languages $L(D_i)$.