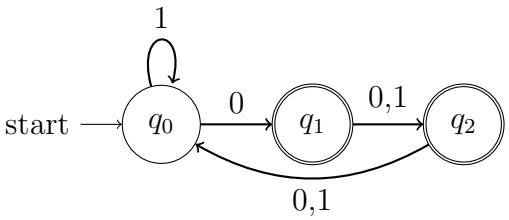
COMS 461 - Midterm 1 Review

1. (8 points) Consider the DFA shown below.



   (a) What sequence of states will this DFA enter as it reads the string 110101?

   (b) Will this DFA accept the string 110101?

2. (16 points) The following statements are all false. For each one, explain why it is false.

   (a) The cardinality of $\{0,1\}^*$ is uncountable.

   (b) The NAND function is universal which means that any function $f : \{0,1\}^* \to \{0,1\}$ can be expressed using NAND functions.

   (c) The union of any two languages $A, B \subset \{0,1\}^*$ is a regular language.

   (d) Boolean logic circuits, DFAs, NFAs, and regular expressions are all equivalent computationally. They are all able to recognize regular languages.

3. (12 points) Let $L \subset \{0,1\}^*$ be the language that contains all strings with at least one 1 and at most one 0. Construct a DFA that accepts L.

4. (16 points) Consider the regular expression $(00)^*1(0|1)$.
   (a) Describe in words the set of strings that this regular expression will match.

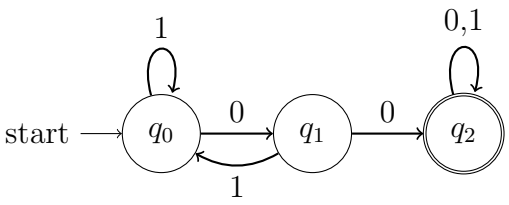
   (b) Construct an NFA (or DFA) that accepts exactly that set of strings.

5. (8 points) In biology, strings of three DNA nucleotides (called *codons*) are known to encode 20 different amino acids. Here, the alphabet consists of the four DNA nucleotides $\Sigma = \{A, C, G, T\}$. Let $\mathcal{A}$ denote the set of 20 possible amino acids.

   (a) How many possible strings of three nucleotides are there? In other words, find $|\Sigma^3|$.

   (b) How many possible functions are there from $\Sigma^3 \to \mathcal{A}$?

6. (8 points) Consider the DFA shown below.



   (a) This DFA can be described by a quintuple $(Q, \Sigma, \delta, q, F)$ where $\Sigma = \{0, 1\}$. What are Q, q and F in this notation?

   (b) Find a regular expression that matches the same set of strings that this DFA accepts.

7. (12 points) Let $L \subset \{0,1\}^*$ be the language consisting of all strings with more 0's than 1's. Use the pumping lemma to prove that L is not regular.

8. (20 points) Let $\Sigma = \{0, 1, +, =\}$ and let

$$L = \{x = y + z : x, y, z \text{ are binary integers, and } x \text{ is the sum of } y \text{ and } z\}.$$

(a) Which of the following strings are in L? Circle all correct answers, there might be more than one.

A. `100=1+10`

B. `11=10+1`

C. `2=1+1`

D. `1000=111+1`

E. `x=11+z, if y=11`

(b) Is it possible to write a regular expression over Σ that represents all valid strings in L? Explain why or why not.