Rules of Algebra

Addition Rules

Neither the order you write terms in a sum nor the order you add them matters:

1. **Associative** $a+(b+c) = (a+b)+c$
2. **Commutative** $a+b = b+a$
3. **Additive Inverses** are negatives:

$$a - a = a + (-a) = 0$$

Multiplication Rules

Neither the order you write factors in a product nor the order you multiply them matters:

1. **Associative** $a(bc) = (ab)c$
2. **Commutative** $ab = ba$
3. **Multiplicative Inverses** are reciprocals:

$$\frac{a}{a} = a\left(\frac{1}{a}\right) = 1$$

Distributive Law

$$a(b+c) = ab + ac$$

- There are no extra rules for subtraction and division. Subtraction is just addition by negatives. Division is just multiplication by reciprocals.

Terms are numbers and expressions that are being added/subtracted.

Factors are numbers and expressions that are being multiplied/divided.

- Distribution expands factors into terms.

$$a(b+c) = ab + ac \qquad (x+2)(x+3) = x^2 + 5x + 6$$

- Factoring un-distributes terms back into factors.

$$a(b+c) = ab + ac \qquad (x+2)(x+3) = x^2 + 5x + 6$$

- Factors cancel in fractions.

$$\frac{\cancel{(x-3)}(x+5)}{(x+1)\cancel{(x-3)}} = \frac{(x+5)}{(x+1)} \quad \text{and} \quad \frac{\cancel{4}\cancel{x}y}{\cancel{12}x^{\cancel{2}}} = \frac{y}{3x}$$

- **(Common mistake)** Don't cancel terms in fractions! Only cancel numbers and expressions that are common factors of both the top and bottom.

$$\underbrace{\frac{4x+3}{4x+7} \neq \frac{3}{7}}_{\text{can't cancel the } 4x \text{ term}} \quad \text{and} \quad \underbrace{\frac{4x+3}{4x+7} \neq \frac{x+3}{x+7}}_{\substack{\text{can't cancel the 4 either} \\ \text{4 isn't a factor of the top or bottom}}}$$

Fraction Rules

Addition/Subtraction

Need a common denominator to add:

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}.$$

You can multiply the top and bottom of each term by a missing factor to get a common denominator:

$$\frac{a}{c} + \frac{b}{d} = \frac{a\color{red}{d}}{c\color{red}{d}} + \frac{b\color{red}{c}}{d\color{red}{c}} = \frac{ad+bc}{cd}.$$

Multiplication/Division

Fractions play nice with multiplication:

$$\left(\frac{a}{b}\right)\left(\frac{c}{d}\right) = \frac{ac}{bd}.$$

Division is just multiplication by the reciprocal of the bottom:

$$\frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)} = \left(\frac{a}{b}\right)\left(\frac{d}{c}\right) = \frac{ad}{bc}.$$

Exponent Rules

Powers represent repeated multiplication

$$a^m = \underbrace{a \cdot a \cdot a \cdots a}_{m\text{-copies}}.$$

Therefore these rules are true:

1. $a^0 = 1$
2. $(a^m)(a^n) = a^{m+n}$
3. $\dfrac{a^m}{a^n} = a^{m-n}$
4. $(a^m)^n = a^{mn}$
5. $(ab)^n = a^n b^n$
6. $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$

Negative Powers

Negative powers are reciprocals.

$$a^{-n} = \frac{1}{a^n}$$

Fractional Powers

Radicals are fractional powers.

$$\sqrt[n]{a} = a^{1/n}$$

- Powers distribute to factors.

$$(ab)^3 = a^3b^3 \qquad \text{and} \qquad \sqrt{9x^2} = \sqrt{9}\sqrt{x^2} = 3x$$

- **(Common mistake)** Powers do not distribute to terms!

$$(a+b)^3 \neq a^3 + b^3 \qquad \text{and} \qquad \sqrt{9+x^2} \neq 3 + x$$

- **(Common mistake)** Negative powers don't make numbers negative. And fractional powers don't mean the result is a fraction. Don't confuse powers with multiplication!