Homework 11 - Math 140 **Name:** ______________________

1. Suppose that a business has total costs of $C(x) = 2x^2 + 3x + 5$ and revenue $R(x) = 5x - 2x^2$. What level of production x would maximize profit (recall that profit is revenue minus costs)?

Find the following differentials.

2. $y = x^3$. Find dy when $x = 2$ and $dx = 0.1$.

3. $Q = 400 - x^2$. Find dQ when $x = 100$ and $dx = 0.5$.

4. $y = x^3 + e^x$. Find dy when $x = 1$ and $dx = 2$.

5. $z = y^5 e^y$. Find dz when $y = 0$ and $dy = 0.1$.

6. $y = \dfrac{20}{x}$. Find dy when $x = 2$ and $dx = -0.1$.

7. $y = \sqrt[3]{x}$. Find dy when $x = 8$ and $dx = 1$.

8. Use the differential in problem 6 to estimate $\frac{20}{1.9}$. Hint: How much larger is $\frac{20}{1.9}$ than $\frac{20}{2}$ according to the dy you found?

9. Use the differential in problem 7 to estimate $\sqrt[3]{9}$. Hint: How much larger is $\sqrt[3]{9}$ than $\sqrt[3]{8}$?

10. The quantity of demand Q is a function of price p. The price elasticity of demand is

$$E = \left| \frac{pQ'(p)}{Q(p)} \right|.$$

Suppose that a business estimates that their customers will purchase $Q(p) = 400 - 10p$ items, when the price of each item is p dollars. What is the elasticity of demand when $p = \$10$?

The relative rate of change in a function $Q(x)$ is $\frac{Q'(x)}{Q(x)}$.

11. The population of a town is $P(t) = t^2 + 200t + 10{,}000$ where t is the time in years since 2020. Find the relative rate of change in P when $t = 5$. Express your answer as a percentage.