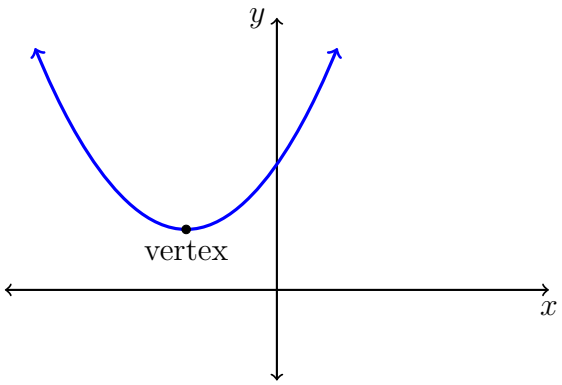
All About Parabolas — Math 140 - Workshop

The graph of a quadratic polynomial $y = ax^2 + bx + c$ is always a **parabola**.



Things to Know

1. The leading coefficient tells you if the parabola is right-side up like the one in the picture (when a is positive) or if the parabola is upside down (when a is negative).
2. The x-values where a parabola crosses the x-axis are called the **roots**.
3. If you can't factor a quadratic, then use the **quadratic formula** to find its roots.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

4. The **vertex** of a quadratic function is located at $x = -\frac{b}{2a}$. If the parabola has two roots, then the vertex is halfway between the roots.
5. The **discriminant** is the expression $b^2 - 4ac$ (which is inside the square root). If it is positive, the parabola has two roots, if it is zero, the only root is the vertex, and if it is negative, then there are no roots (because you can't take a square root of a negative number). A quadratic polynomial with integer coefficients can only be factored nicely if the discriminant is a perfect square.

Practice

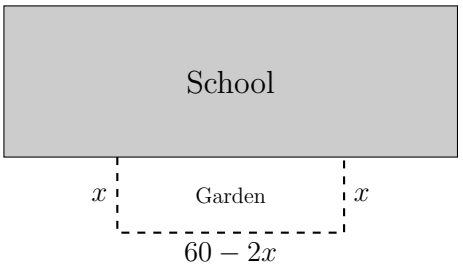
1. For each of the following parabolas, calculate the discriminant $b^2 - 4ac$, and use it to determine if the parabola has zero, one, or two roots.
   (a) $y = x^2 - 6x - 2$

   (b) $f(x) = -4x^2 - 4x - 10$

   (c) $f(x) = 2x^2 - 12x + 18$

2. Find the x-values where the line $y = 2x + 5$ intersects the parabola $y = x^2 - 3$.

3. A local elementary school wants to make a rectangular flower garden along the side of the school building. One side of the garden will the be along the wall of the school, but the other three sides will be fenced off. If you have 60 feet of fencing material, what is the largest area possible for the flower garden?



4. A gas station estimates that if they charge \$2 per gallon, then demand would be 18,000 gallons, but for every dollar they increase the price, the demand drops linearly by 9,000 gallons. Find formulas for the quantity sold Q and the revenue R as a function of the price p. Recall that revenue is price times quantity sold. Then sketch a graph of the revenue function, and find the price where revenue is maximized.

5. The height of a ball thrown in the air is given by $h(x) = -0.1x^2 + 0.7x + 6$, where x is the horizontal distance away from the thrower. Find the x-values of the roots and the vertex of this parabola.