Math 342 Workshop - Error Bounds **Name:** ________________

1. Find the 2nd degree Taylor polynomial for $f(x) = \sqrt{x}$ centered at $c = 4$. Use the following table of derivatives.

k	$f^{(k)}(x)$	$f^{(k)}(4)$
0	$x^{1/2}$	2
1	$\frac{1}{2}x^{-1/2}$	$\frac{1}{4}$
2	$-\frac{1}{4}x^{-3/2}$	$-\frac{1}{32}$
3	$\frac{3}{8}x^{-5/2}$	$\frac{3}{256}$

2. Find an upper bound for the worst case error if you used the 2nd degree Taylor polynomial above to approximate $\sqrt{5}$? Hint: Since $4 \le z \le 5$ in the error formula and $f^{(3)}$ is always decreasing, what value of z would achieve the maximum in Taylor's remainder formula?

3. The function $e^x \approx x + 1$ when x is close to zero. How good is this approximation on the interval $[-1, 1]$? Use Taylor's remainder formula to estimate the worst case error.

4. The **Triangle Inequality** says that for any two numbers a and b,

$$|a + b| \le |a| + |b|.$$

   (a) Use the triangle inequality to show that $|a - b| \le |a| + |b|$. Hint: What does the triangle inequality say about $|a + (-b)|$?

(b) Use the triangle inequality and the fact that $|a \cdot b| = |a| \cdot |b|$ to find an upper bound for $|x^2 \cos x - 3x \sin x|$ on the interval $[0, \pi]$. That is, find a number M such that

$$|x^2 \cos x - 3x \sin x| \leq M$$

for all x in the interval.

5. An extended version of the triangle inequality is also true:

$$|a_1 + a_2 + \ldots + a_n| \leq |a_1| + |a_2| + \ldots + |a_n|$$

for any numbers $a_1, \ldots, a_n$.

(a) Use the extended triangle inequality to find an upper bound for

$$|x^3 - 3x^2 - 3x - 4|$$

on the interval $[0, 2]$.

(b) The graph $y = x^3 - 3x^2 - 3x - 4$ is always negative and decreasing on $[0, 2]$. Can you use that information to get a tighter upper bound for $|x^3 - 3x^2 - 3x - 4|$?

6. The first degree Maclaurin polynomial for the function $f(x) = e^x \sin 2x$ is $P_1(x) = x$. The second derivative of f is $f''(x) = 2e^x \cos 2x - 3e^x \sin 2x$. Use the triangle inequality to get an upper bound for the remainder function on the interval $[-1, 1]$.

$$|R_1(x)| \leq \max_{-1 \leq z \leq 1} \frac{|f''(z)|}{2!} |x|^2.$$