Workshop - Floating Point Operations (FLOPS) **Name:** ________________

Any time a computer uses one of these five operations $(+, -, *, /, \sqrt{\ })$ on floating point numbers, it counts as 1 flop (floating point operation).

1. How many flops does it take to compute the inner product $\mathbf{x} \cdot \mathbf{y}$ for two vectors in $\mathbb{R}^n$?

2. To normalize a vector $\mathbf{x}$ in $\mathbb{R}^n$, you have to compute $\mathbf{x}/\sqrt{\mathbf{x} \cdot \mathbf{x}}$. How many flops does that take?

3. Suppose that $\mathbf{v_1}, \ldots, \mathbf{v_d}$ is an orthogonal basis for a subspace V in $\mathbb{R}^n$. Recall that the orthogonal projection of a vector $\mathbf{x}$ onto V is

$$\text{Proj}_V(\mathbf{x}) = \sum_{k=1}^{d} \frac{(\mathbf{x} \cdot \mathbf{v}_k)}{(\mathbf{v}_k \cdot \mathbf{v}_k)} \mathbf{v}_k.$$

   How many flops does it take to compute $\text{Proj}_V(\mathbf{x})$?

4. It's easier to compute $\text{Proj}_V(\mathbf{x})$ if $\mathbf{v_1}, \ldots, \mathbf{v_d}$ is an orthonormal basis for V. Then the formula simplifies to

$$\text{Proj}_V(\mathbf{x}) = \sum_{k=1}^{d} (\mathbf{x} \cdot \mathbf{v}_k) \mathbf{v}_k.$$

   How many flops does it take to compute $\text{Proj}_V(\mathbf{x})$ with an orthonormal basis?

5. Suppose that A in $\mathbb{R}^{n \times n}$ is an upper triangular matrix. That is,

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ 0 & a_{22} & \ddots & \vdots \\ \vdots & \ddots & \ddots & a_{(n-1)n} \\ 0 & \dots & 0 & a_{nn} \end{bmatrix}.$$

How many floating point operations are required to solve the system $A\mathbf{x} = \mathbf{b}$ using back substitution? Hint: Try to work out the answer for $n = 2, 3$, etc. until you find the pattern.