Math 342 Workshop - Fourier Series **Name:** ________________

The **Fourier series** for a function f in $L^2[-1,1]$ is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos(n\pi x) + b_n \sin(n\pi x)\right)$$

where the **Fourier coefficients** are

$$a_n = \int_{-1}^{1} f(x)\cos(n\pi x)\,dx \text{ and } b_n = \int_{-1}^{1} f(x)\sin(n\pi x)\,dx.$$

1. Find the Fourier coefficients for $f(x) = x$. **Hint:** Use integration by parts to find b_n. Why don't you need to integrate to find a_n?

2. Use Desmos to graph the Fourier series for $f(x) = x$ up to $n = 10$. What is the largest value of x such that the Fourier approximation is equal to $f(x)$ exactly?

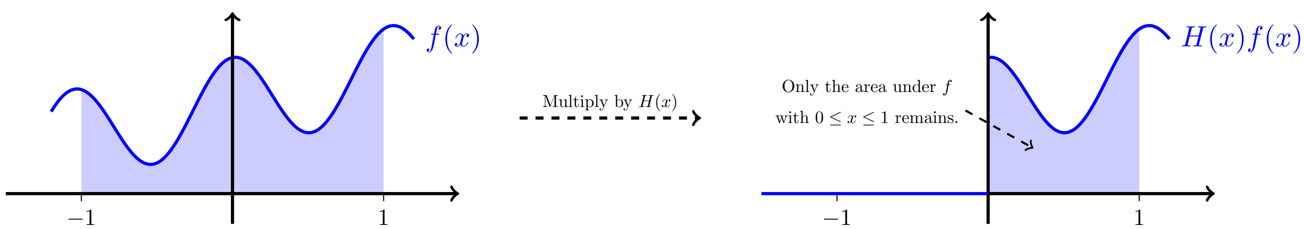
A useful fact about Fourier series is that $\|f(x)\|^2 = \sum_{n=0}^{\infty} a_n^2 + \sum_{n=1}^{\infty} b_n^2$ for any f in $L^2[-1,1]$.

3. Simplify $\sum_{n=1}^{\infty} b_n^2$ using the coefficients b_n from Problem 1 to get one formula for $\|x\|^2$.

4. Use the L^2-norm definition $\|f(x)\|^2 = \int_{-1}^{1} f(x)^2\,dx$ to get a different formula for $\|x\|^2$.

5. **The Basel Problem.** Combine the previous two answers to find $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

6. Let $H(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ 0 & \text{if } x < 0. \end{cases}$ Find the Fourier coefficients a_n and b_n for $H(x)$ up to $n = 10$.
   Hint: To integrate $H(x)f(x)$ for any function f, you only need to integrate f from $x = 0$ to 1.



7. Graph the Fourier series up to $n = 10$ for $H(x)$ on Desmos. What is the maximum y-value for the Fourier series approximation of $H(x)$ according to Desmos?