Math 342 - Numerical Integration **Name:** _____

Write a Python function to apply the midpoint method to any function f. Then use your midpoint method function to answer the following.

1. Recall that $\ln(x) = \int_1^x \frac{1}{x}\,dx$. Use the midpoint method with $n = 1{,}000$ to approximate $\ln(2)$. What is the absolute error in your approximation (to 4 significant digits)?

2. What does the midpoint method error formula give as an upper bound for the error in the previous problem?

3. The function $f(x) = \dfrac{4}{1+x^2}$ has $\max\limits_{0\le\xi\le1} |f''(\xi)| = 8$. In addition,

$$\int_0^1 \frac{4}{1+x^2}\,dx = \pi.$$

   How many midpoint method rectangles would you need, according to the error formula, in order to approximate π with an absolute error less than 5×10^{-11} (i.e., accurate to at least 10 decimal places)?

4. What is the actual absolute error if you use the number of rectangles from Problem 4 to estimate π (accurate to 4 significant digits)?

5. Which is a more accurate way to estimate π using the integral above? Simpson's method with $n = 100$ or the midpoint method with the number of midpoints from Problem 4?