Root Finding Methods **Math 342**

1. The number e is a root of the function $\ln x - 1$. Use Newton's method to approximate e. Start with $x_0 = 3$ and then compute x_1 and x_2. Write your answers to 5 significant digits.

2. Use the secant method with $x_0 = 1$ and $x_1 = 2$ to approximate the root of $x^3 - 5$. Find x_2 and x_3 to 5 significant digits.

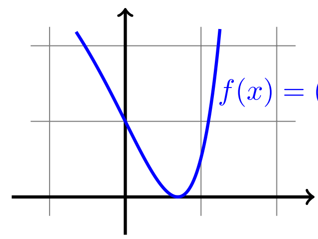
3. Why would $x_0 = 1$ be a bad first guess if you are using Newton's method to find a root of $x^3 - 3x + 6$?

4. The ancient Greek mathematician Hero of Alexandria came up with an iterative algorithm (**Heron's method**) to find the square root of any positive number a. Start with a guess x_0, and then iterate using the formula

$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{a}{x_n}\right).$$

Show that this formula is just Newton's method applied to the function $f(x) = x^2 - a$.

5. The function $f(x) = (e^x - 2)^2$ has one root in the interval $(0, 1)$. Why can't you use the bisection method to find the root?



6. Unlike the bisection method, Newton's method will work to find the root of $f(x) = (e^x - 2)^2$, but the rate of convergence is not fast. If you start with $x_0 = 0$, then even after 10 steps, the approximation x_{10} is only accurate to two decimal places. Can you explain why Newton's method is so slow? Recall the error formula

$$|x_{n+1} - r| \leq \left(\frac{M}{2L}\right) |x_n - r|^2$$

where L is a lower bound for $|f'(x)|$ and M is an upper bound for $|f''(x)|$. Why doesn't that guarantee fast convergence?

7. Let $f(x) = 2x - \cos x$. Find a lower bound L for $|f'(x)|$ and an upper bound M for $|f''(x)|$.