Math 342 Workshop - Nonlinear Systems **Name:** _____

1. Consider the following system of equations.

$$xy = 1$$
$$x^2 - y^2 = 1$$

   (a) Rewrite this system as a vector equation $\mathbf{F}(\mathbf{x}) = \mathbf{0}$. What is $\mathbf{F}$, and what is its Jacobian matrix $\mathbf{J}(\mathbf{x})$?

   (b) Use Newton's method for systems to find a solution to the system above. Write your solution to 5 significant digits. You can graph the equations above on Desmos to check your answer.

2. For any three noncolinear points in a plane, there is exactly one circle that passes through all three points. We'll use Newton's method to find an equation for a circle $(x-a)^2+(y-b)^2 = r^2$ that is intersects the following three points: $(-5, 0)$, $(1, -3)$, and $(4, 2)$.

   (a) Write down a vector equation $\mathbf{F}(\mathbf{x}) = \mathbf{0}$ for these three points with one component of $\mathbf{F}$ for each point and with the vector $\mathbf{x} = \begin{bmatrix} a \\ b \\ r \end{bmatrix}$.

   (b) What is $\mathbf{F}$, and what is its Jacobian matrix $\mathbf{J}(\mathbf{x})$?

   (c) Use Newton's method to find the radius r and the center (a, b) of the circle that passes through all three points above. Write your answer using 5 significant digits. You can check your answer by graphing the circle on Desmos.