Math 342 - Runge-Kutta Method **Name:** ____________________

The fourth order Runge-Kutta method (RK4) is commonly used to numerically approximate the solution of an initial value problem (IVP)

$$\frac{dy}{dt} = f(t, y) \quad \text{with } t \in [a, b] \text{ and initial condition } y(a) = y_0.$$

The formula for updating the y-values in RK4 is

$$y_{i+1} = y_i + \tfrac{1}{6}h(k_1 + 2k_2 + 2k_3 + k_4)$$

where

$$\begin{aligned} k_1 &= f(t_i, y_i), \\ k_2 &= f(t_i + h/2, y_i + hk_1/2), \\ k_3 &= f(t_i + h/2, y_i + hk_2/2), \\ k_4 &= f(t_i + h, y_i + hk_3). \end{aligned}$$

1. The IVP $\frac{dy}{dt} = \frac{y}{t} - \left(\frac{y}{t}\right)^2$ with $y(1) = 1$ has solution $y(t) = \frac{t}{1 + \ln t}$. Use Euler's method to approximate the solution on $[1, 2]$ with $n = 10$ steps. What is the absolute error in your approximation at $t = 2$? Write all answers to 4 significant digits.

2. Write a Python function called RK4 that implements the 4th-prder Runge-Kutta method described above. What is the absolute error when you use the RK4 function to estimate $y(2)$ with $n = 10$?

3. Try different values of n. How low can you get the absolute error?